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## Abstract

The theory of polarization transfer in an inhomogeneous magnetized plasma medium is extended to include the effects of the modecoupling phenomenon. The resultant system of polarization transfer equations is analyzed for several special cases of interest.

#### I. Introduction

Based on the observed polarization properties of certain astrophysical emission, it is desirable to deduce the polarization characteristics of the waves emitted at the source. In such investigations, one has to study the polarization transfer of the radiation through the intervening medium. Generally, two methods are used to study the polarization transfer of e.m. waves (Born and Wolf, 1959; Kawabata, 1964; Kai, 1965; Zheleznyakov, 1968; Fung, 1969): (1) use the system of wave equations and study the spatial transfer of the wave characteristics through the medium; (2) study the transfer of the Stokes parameters. Practically, it is more difficult and tedious to use method 1. The reason for this can be understood after consideration of the different characteristic lengths involved in the two systems of equations appropriate to the two methods; these lengths are the spatial distances required for one periodic variation of the variables considered in each system of equations. The characteristic length of the polarization transfer equations (describing the transfer of Stokes parameters) is the distance of one complete Faraday rotation, which, in most astrophysical plasmas, is much larger than the characteristic length of the wave equations (i.e. the wavelength). If the medium can be considered as "slowly varying" for certain radiation frequency, it is therefore more convenient to study the transfer of polarization properties by means of method 2. Moreover, the additive property of the Stokes parameters enables the simultaneous handling of various phenomena like the differential absorption of the two characteristic modes, emissivity of the medium, etc. However, it should be remarked that the

Stokes parameters are macroscopic quantities, hence any variation of microscopic quantity like the variation in phase velocity (due to mode coupling, for example) can only be deduced by method 1.

The investigation of the transfer of wave polarization in homogeneous magnetized plasmas, including Faraday rotation and differential absorption, was initiated by Kawabata (1964) and Kai (1965). Weak anisotropy of the plasma was included later in the theory of Sazonov and Tsytovich (1968). Zheleznyakov (1968) extended the theory for the case of quasi-longitudinal propagation in a medium with an arbitrary degree of anisotropy. The polarization transfer of radio waves through a magnetized plasma at an arbitrary direction to the static magnetic field has been investigated by Fung (1969). In all these studies, the two characteristic modes (ordinary and extraordinary) are treated as independent during the radiative transfer. However, we know that when electromagnetic waves propagate in an inhomogeneous magnetized plasma, magnetoionic mode coupling can be significant (see Cohen, 1960), and this can affect the polarization properties of the radiation remarkably. It is thus the purpose of this paper to derive a more general system of equations describing the transfer of Stokes parameters at an arbitrary direction in an inhomogeneous magnetized plasma when the effects of magnetoionic mode coupling are included. The calculation error in the derivation of the polarization transfer equations carried out earlier by Fung (1969) is corrected. The system of equation derived is analyzed for some special cases of physical interest.

# II. Derivation of Polarization Transfer Equations

The transfer of electromagnetic radiation through a magnetized plasma will be discussed in terms of Stokes parameters which completely specify the macroscopic polarization properties of the radiation. The Stokes parameters can be defined in terms of the polarization tensor  $\mathbf{I}_{\mathbf{i},\mathbf{j}}$  as

$$I = I_{xx} + I_{yy} \qquad Q = I_{xx} - I_{yy}$$

$$U = I_{yx} + I_{xy} \qquad V = i(I_{yx} - I_{xy})$$
(1)

and the polarization tensor is defined in terms of the components of the electric induction vector as

$$\mathbf{I}_{\mathbf{i},\mathbf{j}} = \langle \mathbf{D}_{\mathbf{i}} \mathbf{D}_{\mathbf{j}} \rangle \tag{2}$$

where < > is the time-average operator. Generally, any electromagnetic wave propagating along the z-direction can be expressed as the sum of two characteristic modes (see Kawabata, 1964):

$$\begin{split} D_{\mathbf{i}}(z,t) &= A_{\mathbf{e}}(z) v_{\mathbf{i}e} \; \exp \{-i [\omega t \; - \int k_{\mathbf{e}}(z) \; dz \; - \; \psi_{\mathbf{e}}] \} \\ &+ A_{\mathbf{o}}(z) v_{\mathbf{i}o} \; \exp \{-i [\omega t \; - \int k_{\mathbf{o}}(z) \; dz \; - \; \psi_{\mathbf{o}}] \} \end{split}$$

where  $A_O(z)$ ,  $A_e(z)$  = wave amplitudes of the o and e modes, respectively  $\psi_e, \ \psi_o = \text{their initial phases}$ 

 $k(\,z) = \widetilde{k}(\,z) \,+\, i\,\varkappa(\,z)$  where the real part  $\,\widetilde{k}(\,z)\,$  is the wave numnumber, and the imaginary part  $\,\varkappa(\,z)\,$  is the amplitudeabsorption coefficient

the subscript i takes the value x, y.

 $v_{ie}, v_{io}$  = components of the polarization vectors expressed in terms of the Appleton-Hartree parameters X,Y, $\theta$ , as given by Kai (1965):

$$v_{xe} = b_1$$
  $v_{ye} = ib_2$  (3)  
 $v_{xo} = ib_2$   $v_{yo} = b_1$ 

where

$$b_{1} = \frac{D + Y_{T}'}{\sqrt{2Y_{L}^{2} + {Y_{T}^{\prime}}^{2} + [DY_{T}/(1-X)]}} \qquad b_{2} = \frac{Y_{L}}{\sqrt{2Y_{L}^{2} + {Y_{T}^{\prime}}^{2} + [DY_{T}/(1-X)]}}$$

$$\sqrt{D = {Y_{T}^{\prime}}^{2} + Y_{L}^{2}} \qquad Y_{T}' = \frac{Y_{T}^{2}}{2(1-X)}$$

$$Y_{T} = Y \sin \theta \qquad Y_{L} = Y \cos \theta$$

The components of the polarization vectors satisfy the following orthogonal relations:

$$\sum_{i} v_{ie} v_{ie}^{*} = \sum_{i} v_{io} v_{io}^{*} = 0$$
 (4a)

$$\sum_{i} v_{ie} v_{io}^{*} = \sum_{i} v_{io} v_{ie}^{*} = 1 \qquad . \tag{4b}$$

When the medium can be considered as slowly varying, the changes

of the amplitudes  $A_0$ ,  $A_e$  in space due to the mode-coupling phenomenon can be expressed as (see Chin and Fung 1970)

$$A'_{o} = C_{e}A_{e} \exp[i\int (k_{e}-k_{o}) dz]$$
 (5a)

$$A'_{e} = C_{o}A_{o} \exp[-i\int (k_{e}-k_{o}) dz]$$
 (5b)

where the coupling coefficients  $C_{o}$ ,  $C_{e}$  and the characteristic polarization  $R_{o}$  are defined as follows:

$$C_{e} = \frac{R'_{e}}{R_{e} - R_{o}} \frac{k_{e}^{3}}{k_{o}^{3}} \exp\left(-\int_{e}^{R'_{o} + R'_{e}} dz\right)$$
 (6a)

$$C_{o} = \frac{R'_{o}}{R_{o} - R_{e}} \frac{k_{e}^{3}}{k_{o}^{3}} \exp \left( \int_{R_{e} - R_{o}}^{R'_{o} + R'_{e}} dz \right)$$
 (6b)

$$R_{o} = \frac{1/2(Y_{x}^{2} - Y_{y}^{2}) \pm [(1/4)Y_{T}^{4} + Y_{L}^{2}(U - X)^{2}]}{Y_{x}Y_{y} - iY_{L}(U - X)}$$
(6c)

where  $Y_x = Y_T \cos \emptyset$ ;  $Y_y = Y_T \sin \emptyset$ ; the prime represents differentiation with respect to z.

For simplicity, a coordinate system  $(\emptyset = 90^\circ)$ ; see Fig. 1) with a magnetic field in the yz-plane is chosen, and the thermal absorption is neglected in  $C_o$  and  $C_e$  in comparison to the wave numbers  $\widetilde{k}_o$  and  $\widetilde{k}_e$ . As a result,  $R_o$  and  $R'_o$  are pure imaginary, as seen from (6c) by setting  $Y_x = 0$ . In this case, the coupling coefficients  $C_o$  and  $C_e$  in Eq. (6) are then real:

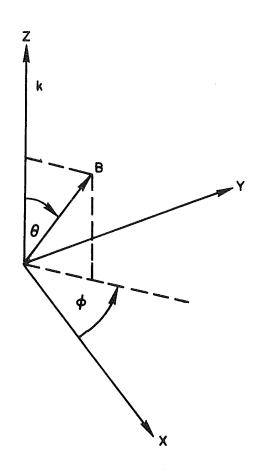


Fig. 1 Coordinate system of the problem

$$c_{o} \simeq \left(\frac{\widetilde{k}_{o}}{\widetilde{k}_{e}}\right)^{3} \frac{|R'_{o}|}{R_{o}^{2} - 1} \qquad c_{e} \simeq -\left(\frac{\widetilde{k}_{e}}{\widetilde{k}_{o}}\right)^{3} \frac{|R'_{o}|}{R'_{o} - 1} \qquad (7)$$

It should be noted that, for the coordinate system with  $\emptyset = 90^{\circ}$ ,  $R_{o} = 1/R_{e}$  and this result has been used for the above derivation.

Following the method of Fung (1969) and Zheleznyakov (1968), we obtain the following equation describing the transfer of the polarization tensor  $\mathbf{I}_{\mathbf{i}\,\mathbf{j}}$ :

$$\frac{\mathrm{dI}_{1j}}{\mathrm{dz}} = -\kappa_{ij1m} I_{1m} + R_{ij1m} I_{1m} + C_{ij1m} I_{1m}$$
(8)

where

$$\begin{split} \varkappa_{\text{ijlm}} &= 2\varkappa_{\text{e}} v_{\text{ie}}^* v_{\text{je}}^* v_{\text{le}}^* v_{\text{me}}^* + 2\varkappa_{\text{e}} v_{\text{ie}} v_{\text{jo}}^* v_{\text{lo}}^* v_{\text{mo}} \\ &+ (\varkappa_{\text{e}}^+ \varkappa_{\text{o}}) (v_{\text{ie}} v_{\text{jo}}^* v_{\text{le}}^* v_{\text{mo}}^* + v_{\text{io}} v_{\text{je}}^* v_{\text{lo}}^* v_{\text{mo}}) \\ R_{\text{ijlm}} &= i (\widetilde{\kappa}_{\text{e}}^- \widetilde{\kappa}_{\text{o}}) (v_{\text{ie}} v_{\text{jo}}^* v_{\text{le}}^* v_{\text{mo}}^* - v_{\text{io}} v_{\text{je}}^* v_{\text{lo}}^* v_{\text{me}}) \\ C_{\text{ijlm}} &= C_{\text{e}} (v_{\text{io}} v_{\text{jo}}^* G_{\text{lm}}^* + G_{\text{ij}} v_{\text{lo}}^* v_{\text{me}}) \\ &+ C_{\text{o}} (v_{\text{ie}} v_{\text{je}}^* G_{\text{lm}}^* + G_{\text{ij}} v_{\text{lo}}^* v_{\text{mo}}) \\ G_{\text{ij}} &= v_{\text{ie}} v_{\text{jo}}^* < \exp[i(\psi_{\text{e}} - \psi_{\text{o}})] > + v_{\text{io}} v_{\text{je}}^* < \exp[-i(\psi_{\text{e}} - \psi_{\text{o}})] > . \end{split}$$

The  $R_{ijlm}$  term characterized the Faraday rotation on the polarization transfer of the waves, and  $\varkappa_{ijlm}$  specifies the effects of absorption on the transfer of intensity and polarization. The changes in the polarization properties of the waves caused by the mode-coupling

phenomenon is contained in the tensor  $C_{ijlm}$ . Because this phenomenon in nonlinear, the Stokes parameters will be nonlinearly related as seen from the definition of  $C_{ijlm}$ . Generally speaking, a source terms  $S_{ij}$  (which describes the contribution to  $I_{ij}$  from the medium) can be added to the right-hand side of Eq. (8). The structure of equation (8) manifests the convenience of the additivity property of the Stokes parameters in handling different phenomena.

The equations describing the transfer of the Stokes parameters now can be derived using the method developed by Zheleznyakov (1968) and by Fung (1969)

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}\mathbf{z}} = -(\kappa_{\mathrm{e}} + \kappa_{\mathrm{o}})\mathbf{I} - (b_{1}^{2} - b_{2}^{2}) (\kappa_{\mathrm{e}} - \kappa_{\mathrm{o}})\mathbf{Q} + 2b_{1}b_{2}(\kappa_{\mathrm{e}} - \kappa_{\mathrm{o}})\mathbf{V} 
+ 2(C_{\mathrm{e}} + C_{\mathrm{o}}) \sqrt{\mathbf{I}_{\mathrm{o}}\mathbf{I}_{\mathrm{e}}} \cos \int (\widetilde{\mathbf{k}}_{\mathrm{e}} - \widetilde{\mathbf{k}}_{\mathrm{o}}) d\mathbf{z}$$
(9a)

$$\frac{dQ}{dz} = -\left(b_1^2 - b_2^2\right) \left(\varkappa_e - \varkappa_o\right) \mathbf{I} + 2b_1 b_2 \left(\widetilde{\kappa}_e - \widetilde{\kappa}_o\right) \mathbf{U} - \left(\varkappa_e + \varkappa_o\right) Q$$

$$- 2\left(b_1^2 - b_2^2\right) \left(C_e - C_o\right) \sqrt{\mathbf{I}_o \mathbf{I}_e} \cos \int \left(\widetilde{\kappa}_e - \widetilde{\kappa}_o\right) dz$$

$$+ 4b_1 b_2 \left(C_e \mathbf{I}_e + C_o \mathbf{I}_o\right) \langle \sin \left(\psi_e - \psi_o\right) \rangle \tag{9b}$$

$$\frac{d\mathbf{U}}{d\mathbf{z}} = -2\mathbf{b}_1 \mathbf{b}_2 (\widetilde{\mathbf{k}}_e - \widetilde{\mathbf{k}}_o) \mathbf{Q} - (\mathbf{n}_e + \mathbf{n}_o) \mathbf{U} - (\mathbf{b}_1^2 - \mathbf{b}_2^2) \mathbf{V} 
+ 2(\mathbf{C}_e \mathbf{I}_e + \mathbf{C}_o \mathbf{I}_o) < \infty \mathbf{s} (\psi_e - \psi_o) >$$
(9c)

$$\frac{d\mathbf{V}}{d\mathbf{z}} = 2\mathbf{b}_{1}\mathbf{b}_{2}(\mathbf{u}_{e}^{-\mathbf{u}_{o}})\mathbf{I} + \left(\mathbf{b}_{1}^{2} - \mathbf{b}_{2}^{2}\right)(\widetilde{\mathbf{k}}_{e}^{-\widetilde{\mathbf{k}}_{o}})\mathbf{U} - (\mathbf{u}_{e}^{-\mathbf{u}_{o}})\mathbf{V} 
+ 4\mathbf{b}_{1}\mathbf{b}_{2}(\mathbf{c}_{e}^{-\mathbf{c}_{o}})\sqrt{\mathbf{I}_{o}\mathbf{I}_{e}}\cos\int(\widetilde{\mathbf{k}}_{e}^{-\widetilde{\mathbf{k}}_{o}})d\mathbf{z} 
+ 2\left(\mathbf{b}_{1}^{2} - \mathbf{b}_{2}^{2}\right)(\mathbf{c}_{e}\mathbf{I}_{e}^{2} + \mathbf{c}_{o}\mathbf{I}_{o}^{2}) < \sin\left(\psi_{e}^{-\psi_{o}}\right) > .$$
(9d)

In the above system of equations the quantities  $\tilde{k}_{e,o}$ ,  $\kappa_{e,o}$ ,  $\kappa_$ 

The initial values of the Stokes parameters can be calculated from Eqs. (2.21) and (2.20) of the paper by Fung (1969).

From Eqs. (9) and (10), we have the transfer equations for  $I_e, I_o$  when the effects of mode coupling are included:

$$\frac{\mathrm{d}\mathbf{I}_{\mathrm{o}}}{\mathrm{d}\mathbf{z}} = -2n_{\mathrm{o}}\mathbf{I}_{\mathrm{o}} + 2C_{\mathrm{o}}\sqrt{\mathbf{I}_{\mathrm{o}}\mathbf{I}_{\mathrm{e}}}\cos\left(\widetilde{\mathbf{k}}_{\mathrm{e}}-\widetilde{\mathbf{k}}_{\mathrm{o}}\right) \,\mathrm{d}\mathbf{z} \quad , \tag{10a}$$

$$\frac{\mathrm{d}\mathbf{I}_{e}}{\mathrm{d}\mathbf{z}} = -2\kappa_{e}\mathbf{I}_{e} + 2C_{e}\sqrt{\mathbf{I}_{o}\mathbf{I}_{e}}\cos\left((\widetilde{\mathbf{k}}_{e}-\widetilde{\mathbf{k}}_{o})\right) d\mathbf{z} . \tag{10b}$$

In view of Eqs. (9) and (10), it can be seen that the phase relationship between the two modes, consisting of the relation between the initial phases and the contributions caused by Faraday rotation, plays an important role in the transfer of the Stokes parameters.

It should also be noted that in the derivation of Eq. (9), a weakly lossy medium (such as the solar corona) has been assumed; therefore, the absorptions  $\kappa_{\rm e}$  and  $\kappa_{\rm o}$  have been neglected in the derivation of the coupling coefficients  $C_{\rm e}$  and  $C_{\rm o}$ . If the medium is significantly lossy (that is, the amplitude absorptions are no longer

negligible in comparison to  $\widetilde{k}_e$  and  $\widetilde{k}_o$ ), the coupling coefficients in Eqs. (7) become complex. In this case, the component fed from one mode to another will have a phase shift dependent on the absorption of the medium.

#### III. Special Cases

In this section, we shall proceed to analyze the system of polarization transfer equations derived in the last section for some special cases of interest. When the effects of coupling are included, the equations describing the transfer of the Stokes parameters are involved and the physical insight of various terms in the equations is not obvious. Therefore, we shall only discuss cases when the mode coupling phenomenon is insignificant.

(a) When the initial phase between waves of the two characteristic modes is finite

In this case  $C_e = C_o = 0$  and system (9) is simplified to

$$\frac{dI}{dz} = -(\kappa_e + \kappa_o)I - (b_1^2 - b_2^2) (\kappa_e - \kappa_o)Q + 2b_1b_2(\kappa_e - \kappa_o)V$$
 (11a)

$$\frac{dQ}{dz} = -(b_1^2 - b_2^2) (\kappa_e - \kappa_o) I - (\kappa_e + \kappa_o) Q + 2b_1 b_2 (\widetilde{\kappa}_e - \widetilde{\kappa}_o) U$$
 (11b)

$$\frac{d\mathbf{U}}{d\mathbf{z}} = -2b_1b_2(\widetilde{\mathbf{k}}_e - \widetilde{\mathbf{k}}_o)\mathbf{Q} - (\varkappa_e + \varkappa_o)\mathbf{U} - (b_1^2 - b_2^2)(\widetilde{\mathbf{k}}_e - \widetilde{\mathbf{k}}_o)\mathbf{V}$$
(11c)

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}\mathbf{z}} = +2b_1b_2(\kappa_e - \kappa_o)\mathbf{I} + (b_1^2 - b_2^2)(\widetilde{\kappa}_e - \widetilde{\kappa}_o)\mathbf{U} - (\kappa_e + \kappa_o)\mathbf{V}$$
 (11d)

Equations (11c) and (11d) are different from the results presented by Fung (1969) because a calculation error has been made in the previous work.

In the above system of equations, the terms associated with coefficients such as  $\pm 2b_1b_2(\widetilde{k}_e-\widetilde{k}_o)$  or  $\pm (b_1^2-b_2^2)(\widetilde{k}_e-\widetilde{k}_o)$  indicate that the magnitudes of Q, U, and V are oscillating at the same

Faraday-rotation rate. The Faraday rotation, therefore, will not simply rotate the whole ellipse but it will also change the eccentricity of its ellipse. The fluctuation of this eccentricity is also effective in the mode-coupling phenomenon, as seen from Eqs. (9). For a homogeneous medium, the solutions to Eqs. (11) are

$$\mathbf{I} = \mathbf{I}_{\mathbf{e}}^{0} \exp(-2\kappa_{\mathbf{e}} \mathbf{z}) + \mathbf{I}_{\mathbf{o}}^{0} \exp(-2\kappa_{\mathbf{o}} \mathbf{z})$$
 (12a)

$$Q = \left(b_{1}^{2} - b_{2}^{2}\right) \left[\mathbf{I}_{e}^{O} \exp(-2\varkappa_{e}z) - \mathbf{I}_{o}^{O} \exp(-2\varkappa_{o}z)\right] + 2b_{1}b_{2}\left[\left(b_{1}^{2} - b_{2}^{2}\right)V^{O}\right]$$

$$+ 2b_{1}b_{2}Q^{O} \exp\left[-\left(\varkappa_{e} + \varkappa_{o}\right)z\right] \cos\left(\widetilde{\kappa}_{e} - \widetilde{\kappa}_{o}\right)z$$

$$+ 2b_{1}b_{2}U^{O} \exp\left[-\left(\varkappa_{e} + \varkappa_{o}\right)z\right] \sin\left(\widetilde{\kappa}_{e} - \widetilde{\kappa}_{o}\right)z \qquad (12b)$$

$$\begin{split} \mathbf{U} &= -\left[\left(\mathbf{b}_{1}^{2} - \mathbf{b}_{2}^{2}\right) \mathbf{V}^{O} + 2\mathbf{b}_{1}\mathbf{b}_{2}\mathbf{Q}^{O}\right] \exp\left[-\left(\mathbf{\mu}_{e} + \mathbf{\mu}_{o}\right)\mathbf{z}\right] \sin\left(\widetilde{\mathbf{k}}_{e} - \widetilde{\mathbf{k}}_{o}\right)\mathbf{z} \\ &+ \mathbf{U}^{O} \exp\left[-\left(\mathbf{\mu}_{e} + \mathbf{\mu}_{o}\right)\mathbf{z}\right] \cos\left(\widetilde{\mathbf{k}}_{e} - \widetilde{\mathbf{k}}_{o}\right)\mathbf{z} \end{split} \tag{12c}$$

$$V = 2b_{1}b_{2}[-I_{e}^{O} \exp(-2\mu_{e}z) + I_{o}^{O} \exp(-2\mu_{o}z)]$$

$$+ U^{O} (b_{1}^{2}-b_{2}^{2}) \exp[-(\mu_{e}+\mu_{o})z] \sin (\widetilde{k}_{e}-\widetilde{k}_{o})z$$

$$+ (b_{1}^{2}-b_{2}^{2}) [(b_{1}^{2}-b_{2}^{2}) V^{O} + 2b_{1}b_{2}Q^{O}] \exp[-(\mu_{e}+\mu_{o})z] \cos (\widetilde{k}_{e}-\widetilde{k}_{o})z$$
(12d)

where  $I_e^0$ ,  $I_o^0$ ,  $V^0$ ,  $U^0$ , and  $Q^0$  are initial values.

In general, if the dominant absorption process is thermal in nature,  $\mu_e \geq \mu_o$ . From equation (3), we observe that  $b_1 \geq 0$  and  $b_1 \geq b_2$ . The consequence of these results is that coefficients of the form  $-(b_1^2-b_2^2)(\mu_e-\mu_o)$  in equations (11) are negative. Since there is

no coefficient of the form  $(\varkappa_e - \varkappa_o)$  appearing in the differential equation for U (i.e. llc), the finiteness of differential absorption will not affect the nature of the variation of U in z. Consequently, the degree of linear polarization  $\rho_L = (Q^2 + U^2)^{-1/2}$  /I will always be reduced as the radiation propagates along. On the other hand, with  $\varkappa_e \ge \varkappa_o$  and  $b_1 \ge 0$ , the sign of the coefficient  $2b_1b_2(\varkappa_e - \varkappa_o)$  (appearing in lla and lld) depends on the sign of  $b_2$ . From equation (3), it is clear that  $b_2$  will assume negative value (positive value) when the angle  $\theta$  between the wave vector and static magnetic field is greater (smaller) than  $90^\circ$ . Hence the value of V and thus the degree of circular polarization  $(\rho_c = |V|/I)$  will be decreased or increased due to the effects of differential absorption (manifests itself in the coefficients  $2b_1b_2(\varkappa_e - \varkappa_o)$ ) depending on whether  $\theta$  is smaller or greater than  $90^\circ$ .

(b) When the phase relation between the two modes is randomized.

When the generating mechanism is incoherent or when a large amount of Faraday rotation occurs in the source region, the phase between the characteristic waves are usually randomized. In this case, the polarization transfer equations are different from the radiation where a definite phase relationship appears between the two characteristic modes. When the phase between the e and o modes is randomized,

$$<\exp[\pm(\psi_e^-\psi_o^-)]>=0$$
.

From Eq. (8),

$$\frac{dI}{dz} = -(\kappa_e + \kappa_o)I - (b_1^2 - b_2^2) (\kappa_e - \kappa_o)Q + 2b_1b_2(\kappa_e - \kappa_o)V$$
 (13a)

$$\frac{dQ}{dz} = \left(b_1^2 - b_2^2\right) \left[ -(\kappa_e - \kappa_o)I - (b_1^2 - b_2^2) (\kappa_e + \kappa_o)Q + 2b_1b_2(\kappa_e + \kappa_o)V \right]$$
(13b)

$$\frac{dV}{dz} = 2b_1b_2 \left[ (\kappa_e - \kappa_o)I + (b_1^2 - b_2^2) (\kappa_e + \kappa_o)Q - 2b_1b_2(\kappa_e + \kappa_o)V \right]$$
(13c)

When the initial phase between the two modes in randomized, the resultant wave after mode coupling gradually will be depolarized as a result of the addition of the components with random phase. No Faraday rotation is observable in this case because all terms related to  $(\tilde{k}_e^- - \tilde{k}_o^-)$  are missing in Eqs. (12). The randomization of the phase between the two modes further reduces the number of independent Stokes parameters; it can be seen from Fung (1969) that U is always equal to zero, leading to the result that the degree of linear polarization is directly proportional to Q.

For longitudinal propagation,  $b_1 = b_2 = 1/2$  which implies that Q = 0. It can be seen from Eqs. (13) that the polarization of the wave can be completely specified by I and V. The nature of polarization can only be circular. For transverse propagation,  $b_2 = 0$  implying V = 0. The only Stokes parameters left are I and Q, so that the polarization is linear. Therefore, for radiation with no initial phase relationship, the nature of polarization is completely determined by the condition of the intervening medium, but not by its generating mechanism at the source.

For a homogeneous medium, Eqs. (12) can readily be solved, and

the solutions are

$$I = I_e^0 \exp(-2\kappa_e z) + I_o^0 \exp(-2\kappa_o z)$$
 (14a)

$$Q = -(b_1^2 - b_2^2) \left[ I_o^0 \exp(-2\mu_o z) - I_e^0 \exp(-2\mu_e z) \right]$$
 (14b)

$$U = O (14c)$$

$$V = 2b_1b_2 \left[ I_o^0 \exp(-2\mu_o z) - I_e^0 \exp(-2\mu_e z) \right]$$
 (14d)

These equations transit to the result of Zheleznyakov, considering  $\textbf{U}^O = \textbf{Q}^O = \textbf{O} \quad \text{in this case.}$ 

#### IV. Conclusions

In this paper, the theory of polarization transfer developed by Kawabata, Kai, Zheleznyakov and Fung has been extended to include the effects of magnetoionic mode coupling. A calculation error in the previous derivation of the polarization transfer equations (Fung, 1969) has been corrected. The system of transfer equations is simplified and analyzed under several physical assumptions. From this study, we observe that if the initial phase relationship between the two modes in finite, Faraday rotation not only rotates the polarization ellipse as the waves propagate, but also changes the eccentricity of the ellipse in an oscillating manner. When the initial phase between waves of the two characteristic modes is randomized, the resulting polarization characteristics of certain radiation passing through a medium are mainly determined by the properties of the intervening medium.

The significance of the mode coupling phenomenon in the observed polarization properties of astrophysical emissions has been discussed by Cohen (1960). The system of polarization transfer equations derived in this paper can therefore be relevant to the study of some types of cosmic and solar radio emissions.

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The theory of polarization transfer in an inhomogeneous magnetized plasma medium is extended to include the effects of the modecoupling phenomenon. The resultant system of polarization transfer equations is analyzed for several special cases of interest.

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